Dynamic Purchase Decisions under Regret: Price and Availability

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Abstract

We model a dynamic purchase context where a consumer is uncertain about the product’s valuation. The consumer has two purchase opportunities for a product: forward purchase in Period 1 and spot purchase in Period 2. Two forms of regret are considered: buyer’s regret for the amount of money a consumer paid in excess of its valuation when buying forward, and hesitater’s regret for the lost opportunity of an increased surplus when not buying forward. We illustrate how regrets affect consumer’s decision: the consumer is more likely to forward buy when he is more averse to hesitater’s regret, whereas when he is more averse to buyer’s regret, the consumer is more likely to delay the decision. In addition, we consider alternative consumer types to characterize how regret affects their spot purchase decisions and what triggers the regret. We show that type inconsistency, i.e., belief of consumer’s future type is different from the actual one, induces an inferior Period 1 purchase decision from the consumer and impairs his resulting expected surplus.

Key Words: regret, consumer behavior, dynamic purchase, type inconsistency

1 Introduction

We study the impact of regret on the consumer’s decision making in a dynamic purchase context. Under regret theory (Bell 1982; Loomes and Sugden 1982; Fishburn 1991), the decision maker compares the option chosen with the foregone option and this comparison can trigger regret (rejoice) if what the decision maker obtains is lower (higher) than what he could have obtained had he made a different choice. Regret theory allows to explain some of the most common behavioral regularities in economics and management, and hence has become a descriptively appealing alternative to expected utility theory (Von Neumann and Morgenstern 1947). Smith (1996) and Yaniv (2000) analyze the role of regret in medical decision making. In economics and finance, Gollier and Salanié (2006) and Muermann et al. (2006) incorporate regret into models of asset pricing and portfolio choice, Braun and Muermann (2004) show that regret can explain the commonly

A consumer might buy early or delay the buying decision until later. While buying late (spot purchase) is done with better information, potential reasons to buy early (forward purchase) include lower price and guaranteed availability. Two forms of regret naturally arise: buyer’s regret, regretting having bought early, or hesitater’s regret, regretting having hesitated buying. We consider the case where the consumer, when deciding whether or not to forward buy, anticipates regret and takes it into consideration (Zeelenberg 1999). Following Engelbrecht-Wiggans and Katok (2008), we assume that the anticipated regret enters additively into consumer’s surplus, and that the effect of regret on consumer’s surplus is proportional to the amount of foregone surplus. Then buyer’s regret refers to the regret due to the amount of money a consumer paid in excess of its valuation and hesitater’s regret refers to the regret from foregone opportunity of an increased surplus. These two forms of regret are consistent with winner’s and loser’s regret in Engelbrecht-Wiggans and Katok (2008). They are also related to psychological findings on action and inaction regret (Gilovich and Medvec 1995), and commission and omission regret (Kahneman et al. 1982). Winner’s, action and commission regret correspond to our buyer’s regret, and loser’s, inaction and omission regret correspond to our hesitater’s regret. Note, however, that while in our paper regret is of dynamic nature, in these other papers it is static.

Our main contribution is twofold. First, we believe that the dynamics of time is essential to the full characterization and understanding of regret. To reflect this, we formulate a parsimonious two-period model which offers new insights into the dynamics of regret in consumer behavior: Buying forward in Period 1 potentially leads to buyer’s regret if the consumer finds out in Period 2 that the product is valued less to him than the price paid; delaying the purchase decision to Period 2 would potentially trigger hesitater’s regret from missing a discount or from facing limited availability. Second, we model alternative consumer types in terms of how regret affects their spot purchase decisions and what triggers the regret, which in turn influences the consumer surplus. Further, we
address the potential type inconsistency, the phenomenon that consumer’s belief of his future type is different from the actual one. The overview paper of Zeelenberg (1999) points out that “Although we have seen that people take regret into account when they know they will experience it, it is still crucial that they make correct predictions of the intensity of their possible future regret. The prediction of future emotions has not been studied extensively.” To our knowledge, we are the first to analyze how inconsistency in belief of type influences decisions as well as emotions in a formal model under regret. We believe our model constitutes a tractable basis for future theoretical and empirical analysis and applications in consumer behavior and management.

2 Model under Regret Neutrality

In this section, we extend the parsimonious two-period model of early order discount in Gundepudi et al. (2001) to also include uncertain availability. The model describes the behavior of a regret-neutral consumer under a dynamic purchase context illustrated in the following two examples.

**Example 1. Early Order Discount** The 50th anniversary INSEAD Summer Ball in 2010 was to be held on May 15 in the Europe campus. Two types of tickets were provided by the organizers: a regular ticket for 210 euros, and an “early bird” ticket for 175 euros that had to be purchased by April 21. Delphine, a young administrative assistant in the Technology and Operations Management department was not sure whether to purchase early, or wait until May 15. This decision depended on the weather risk, whether she had other opportunities (such as party invitations) on that day, which of her friends would be going, and whether she would still have the same boyfriend who was also planning to go.

**Example 2. Uncertain Availability** Tickets for PJ Harvey’s concert in Paris taking place on May 17, 2009 were available three months in advance from Feb 18, 2009. Our colleague Jürgen was thinking of buying a ticket early since he was afraid that they would be sold out quickly, but he was also not sure of whether he could finish on time the submission of a paper important for his tenure and if his friend Dana would also go.

The two examples illustrate that the trade-off between forward purchase and spot purchase is commonly observed in the market for a product whose valuation to the consumer is contingent on future circumstances. Whereas buying forward often comes with a discount of the product and assured availability, buying on the spot allows more time for the consumer to observe the true valuation
of the product. This dynamic purchase context can be stylized as follows. A consumer has two purchase opportunities for a product: forward purchase in Period 1 and spot purchase in Period 2. Consumer’s valuation $V$ for the product is a random variable realized in Period 2, and the price $p$ for the product in Period 2 is exogenously given. If the consumer makes the purchase in Period 1, when facing uncertain $V$, then the forward purchase price is $(1 - z)p$, where the discount $z \in [0, 1)$, and the availability of the product is ensured. If the consumer delays the purchase decision until Period 2, he observes the realization of his valuation for the product and chooses whether or not to buy the product at the spot purchase price $p$, but the product might not be available anymore. We denote the availability of the product in Period 2 as event $A$, where $A = 1$ if the event happens, which has probability $\phi \in (0, 1)$, and $A = 0$ otherwise. We further assume that $V$ and $A$ are uncorrelated.

Define consumer’s surplus as the difference between his valuation of the product and the price paid and assume the consumer is an expected surplus maximizer. In Period 1, the consumer compares the expected surplus of buying forward, $\mathbb{E}_V [V - (1 - z)p]$ with the one derived from delaying the decision until Period 2, $\mathbb{E}_{V,A} [1_A (V - p)^+]$, and therefore decides whether to forward buy or not based on this comparison.\(^1\)\(^2\) The consumer’s Period 1 purchase decision under regret neutrality is summarized in the following lemma. All proofs are provided in the appendix.

**Lemma 1.** An arbitrary regret-neutral consumer buys forward if and only if

$$zp + (1 - \phi) \mathbb{E}_V (V - p)^+ \geq \mathbb{E}_V (p - V)^+.$$ \hspace{1cm} (1)

In the case of guaranteed availability, (1) becomes $zp \geq \mathbb{E}_V (p - V)^+$, and in the case of no forward purchase discount, it becomes $\phi \leq \frac{\mathbb{E}_V (V - p)^+}{\mathbb{E}_V (V - p)^+}$.

Lemma 1 describes the consumer’s trade-off when making forward purchase decision. The terms $zp$ and $(1 - \phi) \mathbb{E}_V (V - p)^+$ denote the discount and ensured availability benefits from buying forward, respectively, whereas the term $\mathbb{E}_V (p - V)^+$ denotes the expected loss of surplus due to not waiting till Period 2 to discover the true valuation of the product.

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\(^1\)Here we denote $a^+ = \max (0, a)$ and $a^- = -\min (0, a)$.

\(^2\)Note that if the consumer chooses not to forward buy, he observes the realization of his valuation $V$ in Period 2 and makes the spot purchase if and only if he gets non-negative surplus, i.e., $V - p \geq 0$. 

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3 Model under Regret

Continuing the examples, in anticipation of how the circumstances would turn out, Delphine ended up not purchasing the “early bird” ticket. On May 15, the sun was shining, Delphine’s friends and the same boyfriend had all bought the “early bird” tickets. However, since Delphine had to pay 35 euros more to get the ticket, she was so affected by the hesitater’s regret that she ended up not going to the ball but rather sitting at home thinking about her miserable situation. Jürgen on the other hand, ended up buying a ticket in advance. However, Dana didn’t get a ticket (sold out) and Jürgen’s paper was not done in time, hence he didn’t go to the concert. Furthermore, the negative emotions of buyer’s regret upset him to such an extent that he anyway was not able to focus on research on that day.

With these examples in mind, we now turn to the description of the model under regret, starting from Period 2. All notations are as defined in Section 2; further, consumer’s surplus now includes the disutility of anticipated regret.

3.1 Period 2

Conditional on the consumer not having bought in Period 1, we consider his spot purchase decision in Period 2. Before proceeding, we describe the consumer’s regret types according to two criteria: how regret affects his spot purchase decision, and what triggers the regret.

Spot Purchase Decision: Pragmatic vs. Regret reflecting We consider two possible types of consumer with respect to his spot purchase decision: pragmatic and regret reflecting. A pragmatic consumer does not let regret affect his decision on whether or not to buy in Period 2 and makes the purchase as long as the valuation of the product exceeds its price, i.e., when $V - p > 0$. A regret reflecting consumer, on the other hand, will only buy in Period 2 if the surplus exceeds the regret of having foregone the discount $zp$ in Period 1. Revisiting the example of Delphine, it appears clear that her decision of not to buy on the spot was not much about the monetary loss of 35 euros but rather about the emotional effects of regret they offset. Had she been pragmatic, she would see that the 35 euros was not a big deal, and her valuation of going to the ball given the perfect circumstances were clearly in excess of 210 euros.

Regret Experience: Buy vs. Anyway We also consider two possible types of consumer in terms of how his experience triggers regret after having made the spot purchase decision: buy and
A buy consumer experiences regret only in the case of actually making the purchase. Hence, upon purchase, his regret is proportional to the amount he overpaid relative to the case of purchasing in Period 1, i.e., proportional to $zp$. An anyway consumer, on the other hand, experiences regret independently of whether or not buying on spot. Hence, if he buys, he would regret over the amount he overpaid, $zp$, and even in the case of not buying, he would regret the positive amount that he could have obtained if he had bought in Period 1, i.e., $(V - (1 - z)p)^+$. The latter being the case of Delphine.

Define $\beta$ and $\eta$ as the intensity of buyer’s and hesitater’s regret, respectively. Let $P$ indicate the case when the consumer is of type pragmatic, $R$ for regret reflecting, $B$ for buy, and $W$ for anyway. A consumer in Period 2 would therefore have four possible combined types: $PB$, $PW$, $RB$, and $RW$. These four types offer a complete characterization of how regret affects consumers’ spot purchase decisions and what triggers the regret. In the interest of parsimony, our model focuses on the extreme possibilities, while a real consumer’s behavior would in general lie in the interior.

The following lemma describes a consumer’s surplus under these four combined types:

**Lemma 2.** A consumer’s surpluses in Period 2 under types PB, PW, RB, and RW are

\[
S_{PB}^2 = 1_{V - p > 0, A = 1} (V - p - \eta pz),
\]
\[
S_{PW}^2 = 1_{V - p > 0, A = 1} (V - p - \eta pz) - 1_{V - p > 0, A = 0} \eta (V - (1 - z)p) - 1_{V - p < 0} \eta (V - (1 - z)p)^+,
\]
\[
S_{RB}^2 = 1_{V - p > \etazp, A = 1} (V - p - \eta pz),
\]
\[
S_{RW}^2 = 1_{V - p > \etazp, A = 1} (V - p - \eta pz) - 1_{V - p > \etazp, A = 0} \eta (V - (1 - z)p) - 1_{V - p < \etapz} \eta (V - (1 - z)p)^+.
\]

These four expressions can be unified in the following formula:

\[
S_2 = 1_{A = 1} (1_{V - p > 1_r \etazp} (V - p - \eta pz) - 1_{V - p < 1_r \etazp} \eta (V - (1 - z)p)^+) - 1_{A = 0} \eta (V - (1 - z)p)^+.
\]

Equation (2) represents a compact form that describes cases of interest in the analysis of consumer dynamic purchase under regret. The next lemma characterizes the relation between surpluses under these four types.

\[\text{Note that a consumer which is pragmatic and buy might actually buy on spot and as a result be worse off than not doing so. While this does not make much practical sense, we still describe it here for completeness.}\]
Lemma 3. For a given $A$, a consumer’s surpluses under type $PB$, $PW$, $RB$, and $RW$ are ordered in the following way for any realization of $V$:

$$S_{RB}^2 \geq S_{PB}^2 \geq S_{PW}^2 \geq S_{RW}^2.$$

Lemma 3 reveals two important points. First, an anyway type consumer is always worse off than a buy consumer because of experiencing regret when not buying. Second, being a regret reflecting type sets a higher threshold for spot purchase so as to avoid the negative surplus when buying, but meanwhile makes it more likely to regret when not buying.

3.2 Period 1

We next consider the consumer’s purchase decision in Period 1. If the consumer purchases in Period 1, he gets the valuation of the product, less the price he paid and the disutility of regret he might have if the valuation of the product turns out to be lower than the price paid. Formally, his surplus is expressed as:

$$S_1 = V - (1 - z)p - \beta(V - (1 - z)p)^-. \quad (3)$$

We assume that the consumer’s valuation $V$ of the product is drawn from a distribution with CDF $F(\cdot)$. Further, we use $\theta$ to denote consumer’s belief about the distribution, such that the higher the $\theta$, the more likely the consumer believes that the product has a high valuation. Formally, such belief is defined as follows.

Definition 1. We define a consumer’s valuation $V \in [l,h]$ drawn from a family of distribution with parameter $\theta$, and CDF $F(\cdot|\theta)$. Without loss of generality, let $\theta \in [0,1]$. Further:

(i) For any given non-negative $u \in (l,h)$, $F(u|\theta)$ decreases as $\theta$ increases.

(ii) $F(u|\theta = 0) = 1$, and $F(u|\theta = 1) = 0$ for any given $u \in (l,h)$.

(iii) $F(h|\theta) = 1$, and $F(l|\theta) = 0$ for any given $\theta \in [0,1]$.

From the definition it follows that consumer’s belief about the valuation of the product is increasing in $\theta$ in a first-order stochastic dominance sense.

In Period 1, consumer cannot observe his Period 2 type, but only has a belief about it. Let such belief be $x \in \{PB, PW, RB, RW\}$. To address the trade-off of whether or not to forward buy, the
consumer compares the expected surplus from purchasing in Period 1, and the one from waiting until Period 2. This difference is formally expressed as following:

\[ EV_A [S_1 - S^2_x], \tag{4} \]

where \( S^2_x \) is described in Lemma 2. Therefore, the consumer forward buys if (4) is positive, and delays the purchase decision if otherwise. The next proposition depicts such consumer’s purchase decision in Period 1.

**Proposition 1.** Given \( \phi \), for a consumer with valuation distribution \( F(\cdot | \theta) \), there exists \( \theta_{RW} < \theta_{PW} < \theta_{PB} < \theta_{RB} \), such that:

(i) Regardless of type, the consumer will wait until Period 2 if \( \theta \leq \theta_{RW} \); and purchase in Period 1 if \( \theta \geq \theta_{RB} \).

(ii) A type \( x \) consumer will purchase in Period 1 if \( \theta \geq \theta_x \); and wait until Period 2 otherwise.

Proposition 1 illustrates how consumer’s decision in Period 1 depends on his belief about the product’s valuation and his type in Period 2. If he expects the product to have a sufficiently high valuation, he would forward buy regardless of his belief about his Period 2 type; similarly, if he expects the product to have a sufficiently low valuation, he would delay the purchase decision. On the other hand, if he expects the product to have a “moderate” valuation, the consumer’s decision actually depends on his belief about his Period 2 type: for each type, there exists a unique threshold level of the product’s valuation, such that once his belief about the value distribution is above this threshold, consumer would purchase in Period 1, and when otherwise, he would wait until Period 2. In particular, given that \( \theta_{RW} \leq \theta_{PW} \leq \theta_{PB} \leq \theta_{RB} \), when consumer believes himself as an anyway type in Period 2, he is more likely to forward buy than as a buy type. The next corollary illustrates how the intensity of consumer’s regret affects his purchase decision.

**Corollary 1.** Given \( \theta \) and \( \phi \in (0, 1) \), \( EV_A [S_1 - S^2_x] \) is increasing in \( \eta \) and decreasing in \( \beta \) for \( x \in \{PB, PW, RB, RW\} \).

Corollary 1 shows that if a consumer would regret more due to buyer’s regret, he is more likely to buy on the spot, whereas if he would regret more due to hesitater’s regret, he is more likely to buy forward. Simonson (1992) suggests that consumers who anticipate how they would feel if they made the wrong move (such as not buying forward), would be more likely to purchase a currently available item on sale rather than wait for a better sale. This corresponds to the first part of our result. Further, the second part of our result predicts that when consumers anticipate how they would feel if they made the wrong move as buying early, they are more likely to wait.
3.3 Type Inconsistency

Zeelenberg (1999) points out that in addition to the importance of anticipated regret for consumers, it is also important to accurately predict their possible future regret. To reflect this aspect, we consider the case where a consumer in Period 1 has incorrect belief about his true Period 2 type. Specifically, we define a type inconsistent consumer as one whose belief of his future type is different from the actual one; on the other hand, a type consistent consumer is one whose belief of his future type is the same as the actual one.

Due to the optimistic bias in self prediction (Armor and Taylor 1998), consumers might think their future-selves to be better behaved, i.e., in our setting more likely to be a pragmatic and buy type rather than a regret-reflecting or an anyway type. Related to this, there exists a stream of literature studying possible sources of error in the prediction of emotions (see Wilson and Gilbert 2003 for a review). Our model offers an attempt to formalize these issues in the context of regret, and it is the first to address the consequence of possessing inaccurate belief about one’s future type in a dynamic purchasing context. Consider a consumer in Period 1 who believes his Period 2 type to be $x'$ whereas his true type then is $x$, where $x, x' \in \{PB, PW, RB, RW\}$, and define his corresponding surplus as $S^{x'.x}$. Then, given his true type $x$, it follows that the consumer is always no worse off being time consistent. Formally, this translates into

$$E_{V,A}\left[S^{x.x} - S^{x'.x}\right] \geq 0.$$  \hspace{1cm} (5)

A consumer faces two types of uncertainties when making a dynamic purchase decision described in this paper. On one hand, he is not sure of his true valuation of the product, such that he does not know whether and when he should purchase. On the other hand, he might have inaccurate belief about his own type, such that he might make an ex-ante wrong decision due to a wrong estimate of his type. Proposition 1 analyzes consumer’s decision rules given the uncertainty of the product’s valuation, and the above analysis reveals the consequences for ignoring the uncertainty of his own future type.

4 Conclusions

Psychologists have long recognized the importance of regret in human decision making. However, the cognitive environments psychologists consider are not typical of consumer behavior and managerial settings in general. In particular, the dynamic aspects of consumer’s decisions under regret
have been largely ignored. This paper aims at characterizing regrets in a consumer dynamic purchase context.

Our main contribution is twofold. First, we believe that the dynamics of time is essential to the full characterization and understanding of regret. To reflect this, we formulate a parsimonious two-period model which offers new insights into the dynamics of regret in consumer behavior: A consumer might regret having bought forward rather than waiting for better information, or might regret delaying the purchase decision and fail to ensure availability or a better price. Second, we model possible types of consumers both in terms of how regret affects their decisions and what triggers the regret, which in turn influences the consumer surplus. Further, given that consumer might not be able to accurately predict his future type (Wilson and Gilbert 2003; Zeelenberg 1999), we study how type inconsistency influences decisions as well as emotions. We believe our model constitutes a tractable basis for future theoretical and empirical analysis and applications in consumer behavior and management. Possible directions include: (i) a model including the use of strategic stock-outs and early-order-discount to stimulate revenues, in part by achieving segmentation of customer classes, (ii) a model taking into consideration the supply quantity decision under regret, (iii) experimental investigation to quantify the absolute and relative magnitude of the different types of regret, and (iv) a model using nonlinear forms of regret as in Bleichrodt et al. (2010). We leave these and further extensions to future work.

References

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Appendix A. Dynamic Purchase under Regret and Rejoice

In this section, we introduce rejoice for the sake of completeness. Further to the introduction in Section 3, we define $\rho_1$ and $\rho_2$ as the intensity of consumer’s rejoice in Period 1 and 2, respectively. We first consider consumer’s spot purchase decision in Period 2. Lemma 4 is an extension to Lemma 2 and 3.

**Lemma 4.** In Period 2, consumers’ surplus under types PB, PW, RB, and RW are

$$
S_{2PB}^p = 1_{V-p>0, A=1} (V-p-\eta z p) + \rho_2 ((1-z)p-V)^+, \\
S_{2PW}^p = 1_{V-p>0, A=1} (V-p-\eta z p) + 1_{V-p>0, A=0} (-\eta (V-(1-z)p))
+1_{V-p<0} (-\eta (V-(1-z)p)^+) + \rho_2 ((1-z)p-V)^+, \\
S_{2RB}^p = 1_{V-p>\eta z p, A=1} (V-p-\eta z p) + \rho_2 ((1-z)p-V)^+, \\
S_{2RW}^p = 1_{V-p>\eta z p, A=1} (V-p-\eta z p) + 1_{V-p>\eta z p, A=0} (-\eta (V-(1-z)p))
+1_{V-p<\eta z p} (-\eta (V-(1-z)p)^+) + \rho_2 ((1-z)p-V)^+,
$$

where $\rho_2$ denotes the intensity of rejoice in Period 2. For a given $A$ and any realization of $V$,

$$
S_{2RB}^p \leq S_{2PB}^p \leq S_{2PW}^p \leq S_{2RW}^p.
$$

Now we turn to consumer’s forward purchase decision in Period 1. If consumer purchases in Period 1, his corresponding surplus becomes:

$$
S_{1P} = V - (1-z)p - \beta (V-(1-z)p)^- + \rho_1 (V-(1-z)p-1_{A=1}(V-p)^+)^+, \\
S_{1R} = V - (1-z)p - \beta (V-(1-z)p)^- + \rho_1 (V-(1-z)p-1_{V>p+\eta z p, A=1}(V-p))^+,
$$

where $\rho_1$ denotes the strength of rejoice in Period 1. Note that, different from surplus under regret only, consumer’s surplus in Period 1 under both regret and rejoice depends on his Period 2 regret type, i.e., whether he is a pragmatic type or a regret reflecting type. The next proposition is an
extension to Proposition 1 which depicts consumer’s forward purchase decision in Period 1 under both regret and rejoice.

PROPOSITION 2. Given $\phi$, for a consumer with valuation distribution $F(\cdot | \theta)$, there exists $\theta_{RW} \leq \theta_{PW} \leq \theta_{PB} \leq \theta_{RB}$, such that

(i) Regardless of his type, the consumer will wait until Period 2 if $\theta \leq \theta_{RW}$; and purchase in Period 1 if $\theta \geq \theta_{RB}$.

(ii) A type $x$ consumer will purchase in Period 1 if $\theta \geq \theta_{x}$; and wait until Period 2 otherwise.

Following Section 3.3, under both regret and rejoice, a type consistent consumer is still no worse off than a type inconsistent one; therefore, (5) continues to hold. The next corollary illustrates how the intensity of consumer’s regret and rejoice affects his purchase decision.

COROLLARY 2. For a given $\theta, \phi \in (0,1)$, $E_{V,A}[S_{1} - S_{2}]$ is increasing in $\eta$ and $\rho_1$, and decreasing in $\beta$ and $\rho_2$.

To sum up, the introduction of rejoice, though adding some complication into consumer’s surpluses, does not qualitatively change the results with regret only. In particular, consumer’s purchase decision in Period 1 remains the same (Proposition 2), a type consistent consumer is still better off, and the intensity of consumer’s regret affects his purchase decision in the same direction (Corollary 2).

Appendix B. Proofs

PROOF OF LEMMA 1: Consumer would forward buy if and only if $E_{V} [V - (1 - z) \hat{\leq} E_{V,A} [1_{A} (V - p)^+]$. Noticing that $V - p = (V - p)^+ - (p - V)^+$, (1) follows directly. The expression for guaranteed availability follows from setting $\phi = 1$ and the expression for no forward purchase discount from $z = 0$. □

PROOF OF LEMMA 2: Following the definition of four regret types, for consumer of type PB, when $V - p > 0$ and the product is available, he purchases and regrets over the price difference $zp$; when $V - p < 0$ or the product is not available, the consumer does not purchase and does not regret; the expression for $S_{2}^{PB}$ follows. For consumer of type RB, the surplus is the same as for PW except for the purchase threshold $V - p > \eta z p$. For consumer of type PW, when $V - p > 0$ and the product is available, he purchases and regrets over the price difference $zp$; when $V - p < 0$ or the
product is not available, the consumer does not purchase but still regrets over the foregone surplus \( V - (1 - z) p \); the expression for \( S_{2}^{PW} \) follows. For consumer of type RW, the surplus is the same as for PW except for the purchase threshold \( V - p > \eta z p \). This concludes the proof. \( \square \)

**Proof of Lemma 3:** Consider \( A = 1 \) and \( A = 0 \), respectively. Lemma 3 follows directly from Lemma 2. \( \square \)

**Proof of Proposition 1:** First, we need to show that \( \mathbb{E}_{V,A} [S_1 - S_2^x] \) is non-decreasing in \( \theta \), where \( x \in \{PB, PW, RB, RW\} \). For a given \( A \), it is straightforward that \( S_1 - S_2^x \) is non-decreasing in \( V \). Let \( V \) and \( V' \) be consumer’s valuations drawn from two different families of distribution with parameter \( \theta \) and \( \theta' \), respectively. By Definition 1 part one, for \( \theta \leq \theta' \), \( V \leq_{st} V' \). By characterization of first order stochastic dominance (Müller and Stoyan 2002), \( V \leq_{st} V' \) if and only if \( \mathbb{E}_{V} [\psi (V)] \leq \mathbb{E}_{V'} [\psi (V')] \), where \( \psi (\cdot) \) is a non-decreasing function. Therefore, for a given \( A \), \( \mathbb{E}_{V} [S_1 - S_2^x] \) is non-decreasing in \( \theta \). Given that \( \mathbb{E}_{V,A} [S_1 - S_2^x] = \phi \mathbb{E}_{V,A=1} [S_1^x - S_2^x] + (1 - \phi) \mathbb{E}_{V,A=0} [S_1^x - S_2^x] \), \( \mathbb{E}_{V,A} [S_1^x - S_2^x] \) is also non-decreasing in \( \theta \). Then, at the boundaries, by Definition 1 part two, we have \( \mathbb{E}_{V,A} [S_1 - S_2^x] |_{\theta = 1} > 0 \) and \( \mathbb{E}_{V,A} [S_1 - S_2^x] |_{\theta = 0} < 0 \).

On the other hand, by Lemma 2, for any realized \( A \) and \( V \),

\[
S_1 - S_2^{RW} \leq S_1 - S_2^{PW} \leq S_1 - S_2^{PB} \leq S_1 - S_2^{RB},
\]

further, we have

\[
\mathbb{E}_{V,A} [S_1 - S_2^{RW}] \leq \mathbb{E}_{V,A} [S_1 - S_2^{PW}] \leq \mathbb{E}_{V,A} [S_1 - S_2^{PB}] \leq \mathbb{E}_{V,A} [S_1 - S_2^{RB}].
\]

Therefore, there exists \( 0 \leq \theta_{RW} < \theta_{PW} < \theta_{PB} < \theta_{RB} \leq 1 \) such that \( \mathbb{E}_{V,A} [S_1^x - S_2^x] \leq 0 \) when \( \theta \leq \theta_{x} \), and \( \mathbb{E}_{V,A} [S_1^x - S_2^x] > 0 \) if otherwise. This concludes the proof. \( \square \)

**Proof of Corollary 1:** From Lemma 2, for a given \( x \in \{PB, PW, RB, RW\} \), \( \mathbb{E}_{V,A} [S_2^x] \) is decreasing in \( \eta \). On the other hand, from (3), \( \mathbb{E}_{V,A} [S_1] \) is decreasing in \( \beta \). Therefore, \( \mathbb{E}_{V,A} [S_1 - S_2^x] \) is increasing in \( \eta \) and decreasing in \( \beta \). \( \square \)